GRAVITATING WAVES.

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In my various discussions of luminous and gravitating harmonies, I have shown many slight discrepancies, between theoretical and observed results, which are of the same order of magnitude as planetary orbital eccentricities. Although it would be unreasonable to look for any speedy and complete solution of those discrepancies, I think it right to try such questionings of nature, as seem likely to lead to a fuller understanding of the common laws of molar and molecular force.

The hypotheses of Newton and Le Sage seem necessarily to involve a repellent action of the æthereal waves between two bodies or particles, as well as a centripetal appulsion by the exterior waves. If the ratio of these activities is discoverable, it seems reasonable to look for it in the relative positions and motions of the three controlling bodies in the principal subdivisions of our system,—Sun, Earth, and Jupiter.

In the simplest form of gravitating or other central revolution, the tangential "lines of force" are continually deflected, by radial centripetal waves, so as to form a system of semi-circular undulations. The velocity of circular orbital motion communicated by any central force being represented by radius, (or twice the virtual centripetal appulsion), the length of the aggregating radial wave : the length of the deflected semi-circular wave : : 1 : π. But the length of the wave of dissociation* : the length of the limiting wave of aggregation : : 2 : π^2. Combining these proportions, we find that the length of the dissociating or repelling wave : the length of the primitive wave : : 2 : π^3, or : : .0645 : 1. If the repulsion of the surfaces of two bodies from their common centre of gravity is $\frac{2}{\pi^3}$ of the appulsion towards the centre of gravity, the distance of the common centre of gravity from the principal centre of mass $= \left(1 + \frac{2}{\pi^3}\right) r = 1.0645\ r$.

The mean distance of Jupiter from Sun being 1117.87 r, the mass of (Sun $\div$ Jupiter) should be, to accord with this hypothesis, 1117.87 $\div$ 1.0645 = 1050.14.

I have already shown that the limit of dissociating velocity (v_0) for Jupiter and Earth, corresponds to the limit of planetary velocity for Sun, thus indicating an equality of radial and tangential action, such as we might reasonably have anticipated. If we adopt Cornu's determination of the

* Proc. Am. Assoc., Hartford Meeting, 1874; Am. Jour. Sci., "Velocity of Primitive Undulation," Nov. 1874.

velocity of light, so as to derive all our data from observations which are always susceptible of verification, we find the following accordances.

I. For Earth :

$$v_2 = \frac{2}{\pi} v_\rho = \frac{4r}{t} = \frac{15851}{24} \qquad = \qquad 660^m.46 \text{ per } h.$$

$$v_1 = \sqrt{2\,gr} = \sqrt{\frac{32 \times 7925.5}{5280}} \times 3600 \qquad = \qquad 24,950^m.2 \quad \text{" "}$$

$$v_0 = \frac{gt}{2} = \frac{v_1^2}{v_2} = \frac{32 \times 43200 \times 3600}{5280} \qquad = \qquad 942,545^m \quad \text{" "}$$

$$v_0 \div \sqrt{214.86} = \text{approx. Solar } \sqrt{gr} \text{ at } \oplus \qquad = \qquad 64,302^m \quad \text{" "}$$

Multiplying by 8766^h and dividing by 2π we get, for an approximate estimate of Sun's distance, $89,711,000^m$ (α).

The distance corresponding to Cornu's estimate of the Solar parallax ($8''.86$) is $(206264.81 \div 8.86) \times 3962.75 = 92,255,000^m$ (β).

Dividing (β) by (α) we obtain 1.0284, which is nearly equivalent to $\sqrt{1.0645}$.* Therefore v_0 for Earth is nearly, if not precisely, equivalent to planetary velocity in a circular orbit at the centre of gravity of Sun and Jupiter.

II. For Jupiter :

The uncertainty of the elements in this case precludes the possibility of any minute verification of hypothesis, but it is evident that the point at which the gravitating waves must act, in order that the dissociating velocity of Jupiter $\left(v_0 = \frac{gt}{2}\right)$ may equal the limit of planetary velocity, must be at or near Jupiter's surface. For the mass of (Jupiter $\div$ Earth) $= \frac{1}{1050.14} \div 4.432 \left(\frac{8.86}{1000}\right)^3 = 308.92$. The apparent diameter of Jupiter is variously estimated, from $3'\,13''$ to $3'\,25''.5$ at Earth's mean distance from Sun. Dividing by $2 \times 8''.86$, we find for diameter (♃ $\div$ $\oplus$) 10.89 @ 11.60, and for g (♃ $\div$ $\oplus$) 2.3 @ 2.6. The estimates for the time of rotation (t) vary between $17700^{sec.}$ and $17880^{sec.}$

Therefore :

$v_0 = \frac{gt}{2} = gt\,\frac{(32 \times 3600)}{5280} = 888,219$ @ $1,014,283^m$ per h. The geometrical mean of these possible extreme values, differs from the value found for Earth by only 7–10 of one per cent. The other planets, both of the Jovian and of the Telluric belt, would all be dissipated and absorbed in their primaries before they had attained the dissociating velocity of Jupiter and Earth. This intimate dependence of planetary aggregation, dissociation, and rotation, upon Solar attraction, and the dependence of Solar aggregation, dissociation, rotation, and planetary revolution upon

* $\sqrt{1.0645} = 1.0317$.

the velocity of light, therefore point to the same unity of force as has been indicated by the modern researches in heat, electricity, and magnetism.

III. For light and Terrestial Gravity :

If g = equatorial gravitating velocity, and t = a sidereal year, $gt \div \sqrt{1+\frac{2}{\pi_3}} = 365.256 \times 86400 \div 5280 \div \sqrt{1.0645} = 185,380$ m per second. This corresponds to the velocity of light, giving a Solar distance of $497.83 \times 185,380 = 92,287,700$ miles.

VI. Wave Lengths :

The primary radius, $1.0645 \times 92,255,000 \times 63360 \div 214.86 = 28,959,800,$-000 inches. Dividing by the number of wave-lengths* in radius, 66456 (10,[11], we find for the value of one wave length, $u = \frac{1}{229131}$ *in.* The radial waves should be accompanied by deflected tangential waves of three kinds, viz. :

1. $w_2 = \frac{\pi}{2} u = \frac{1}{145869}$ *in.* = wave of simple rotation.

2. $w_1 = \pi u = \frac{1}{72934}$ *in.* = wave of circular orbit.

3. $w_0 = 2\pi u = \frac{1}{36467}$ *in.* = wave of virtual fall *doing work* = Solar orbital wave = 4 w_2.

According to Eisenlohr,† the wave-lengths in the diffraction spectrum are as follows :

Upper actinic,	$\frac{1}{143880}$ *in.*
Lower actinic, or upper luminous,	$\frac{1}{71940}$ *in.*
Lower luminous, or upper thermal,	$\frac{1}{35970}$ *in.*

V. Miscellaneous :

Among other note-worthy accordances in this connection are the following :

1. The approximate equality of Mass (♃ ÷ ☉) to distance fallen through in (time of fall to centre ÷ time of circular revolution).

2. The equality of orbital *vis viva* in Jupiter and Saturn.

3. The equality in the ratio of orbital *vis viva* (♆ ÷ ♅) to the ratio of orbital to radial waves ($w \div u$).

4. The connection of Sun's radius, modulus of light, and the limits of the planetary system ; the velocity of planetary revolution and Solar rotation being equal at 37 ☉ ; v. of revolution at 37 ♆ $\left(=\frac{M}{2}\right)$ = v. of rotation at ☿.

5. The stellar-solar parabola, between α Centauri and Sun, and its relations to the planetary distances.

* *Loc. cit.* † Am. Jour. Sc. [2] xxii, 400.